

CONCOURS D'ENTREE EN 1ère ANNEE – SESSION OCTOBRE 2020

EPREUVE DE MATHEMATIQUES

Duration 3h00 - Coefficient 4

EXERCICE 1 : 3 Points

In an assembly of 250 persons, there are 120 men with black trousers, 85 men with tie of which 50 are in black troussers. A person is chosen at random from the assembly for an interview.

- | | |
|--|--------------|
| 1) What is the probability that he is in a black trouser? | 0.5pt |
| 2) What is the probability that he is in a black trouser and has a tie? | 0.5pt |
| 3) What is the probability that he is in black trousers or has a tie? | 1pt |
| 4) What is the probability that he wears neither a black trouser nor a tie? | 1pt |

EXERCICE 2 : 5 Points

The municipal authority of a town in Cameroon envisage to construct low cost houses for municipal councilors. In order to fix the lodging price, the council accountant reveals the salaries X_i and rent proposition Y_i of a sample of eight agents. The results expressed in thousands of francs CFA are shown on the table below.

X_i	50	100	60	120	120	100	150	160
Y_i	15	20	15	30	25	25	40	35

The table above defines a statistical series with double character (X, Y)

Where X is the salary and Y the rent proposition.

- 1)** a) Plot a scatter diagram representing the clouds of the series. **1pt**
- b) Determine the coordinate of the mean of the series. **1pt**
- 2)** a) Calculate the coefficient of linear correlation of the series. **1pt**
- b) Can the salaries permit to explain the rent proposition? **0.5pt**
- 3)** Use the least square method to determine the linear regression line of Y on X **1.5pt**

EXERCICE 3 : 4 Points

- 1)** Write in a simpler form $c = \ln \frac{4}{9} + \frac{1}{2} \ln 36 + \frac{2}{3} \ln \frac{27}{8}$. **0.5pt**
- 2)** Express in the form $X+Yi$, the complex numbers: **1.5pt**

$$Z_1 = 4(-2 + 3i) + 3(-5 - 8i) \quad \text{and} \quad Z_2 = \left(\frac{1+i}{2-i}\right)^2 + \frac{3+6i}{3-4i}$$

- 3)** Solve the equation: $z^3 + (1+i)z^2 + (i-1)z - i = 0$.
- a) Given that one of the roots is purely imaginary. **0.5pt**
- b) Determine real numbers a and b such that :
- $$z^3 + (1+i)z^2 + (i-1)z - i = (z - ai)(z^2 + bz + c). \quad \text{0.5pt}$$
- c) Deduce all the roots of the equation. **1pt**

EXERCICE 4 : 8 Points

Consider the function of real values f defined on IR by: $f(x) =$

$$\begin{cases} 2 - e^x & \text{if } x < 0 \\ 1 + \ln(1 + x) & \text{if } x \geq 0 \end{cases}$$

And (C_f) is the curve representing f in an orthonormal reference frame

- 1)** Determine the domain of definition of f **0.5pt**
- 2)** Study the continuity and the differentiability f at 0 **1pt**
- 3)** Write the equation of the tangent to (C_f) at the point $x=0$ **0.5pt**
- 4)** Calculate the limits of f at $x=-\infty$ and at $x = +\infty$ **0.5pt**
- 5)** Study the infinite branches of the curve (C_f) . **0.5pt**
- 6)** Study the variations and draw a table of variation of f **1pt**
- 7)** Sketch (C_f) after sketching the tangent to (C_f) at the point $x = 0$ **1.5pt**
- 8)** Using integration by parts, evaluate $\int_0^1 \ln(1 + x) dx$ **0.5pt**
- 9)** Given that K is the area of the domain limited by (C_f) , the $x - axis$ and the lines with equations $x = -\ln 2$ and $x = 1$
Write K in an integrable form and calculate K . **1pt**
- 10)** Show that the restriction of g of f in the interval $[0; +\infty[$ is a bijection of $[0; +\infty[$ in an interval J and determine J . **0.5pt**
- 11)** Show that the equation $g(x) = x$ has a unique solution α in the interval $1 \leq \alpha \leq 3$ **0.5pt**